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ATOMIC DECOMPOSITION VIA PROJECTIVE GROUP REPRESENTATIONS

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ABSTRACT. In the last few years, representations of functions (or distributions) as sums of "building blocks" have attracted much attention, e.g., time-frequency analysis and wavelet analysis. From a more abstract point of view, the Feichtinger/Gröchenig theory discuss the same problem, with an integrable group representation as the starting point. Here we present a survey of the FG-theory combined with a generalization to projective representations; this makes it directly applicable to Gabor analysis. Furthermore we point out the connections to the existing theory for frame decomposition.

1. Introduction. Consider an integrable representation $(\pi, \mathcal{H})$ of a locally compact group $\mathcal{G}$. It is natural to settle the question of whether there exists a discrete family $\{x_i\}_{i\in I} \subseteq \mathcal{G}$ and $g \in \mathcal{G}$ such that any $f \in \mathcal{H}$ can be written as a superposition of the "building blocks" $\{\pi(x_i)g\}_{i\in I}$.

The Feichtinger-Gröchenig theorem [**6, 7, 11**] gives an answer to this question, not only for elements f in $\mathcal{H}$, but also for elements in the so-called *coorbit spaces*. Thus, FG-theory can be considered as generalized wavelet analysis, and as such it deserves to be known among the wavelet experts. Here we present a survey of the theory, combined with an extension to projective representations. This generalization shows how the theory works and should give the reader more feeling with it. And the generalization is not only of theoretical interest but also important in applications. For example, in $L^2(\mathbf{R})$ building blocks arising by translating and modulating a fixed function are of great importance, leading to the important time-frequency decomposition (discussed, e.g., in [**2, 5, 12**]). Unfortunately, the two operations do not compose to a representation (only to a projective representation). Earlier this problem was solved by extending with a torus component to get the Schrödinger representation, but with the present generalization the theory is directly applicable.

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