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DIFFERENTIAL FORMS ON MODULAR CURVES $\mathbf{H}/\Gamma(k)$

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ABSTRACT. In this paper we continue the investigation of the geometric properties of modular curves using θ functions with rational characteristics started in [**4**] and [**2**]. In those papers, θ functions with rational characteristics were used to construct $SL_2(Z_k)$ equivariant mapping $\mathbf{H}/\Gamma(k) \to \mathbf{CP}^{(k-3)/2}$. Moreover, quotients of modular curves were also included in [**1**]. Recently, Farkas and Kra used the theory developed in the cited papers to give new proofs of Ramanujan's congruences and discover some new ones, see [**5**] for details. In the present paper we construct differentials on holomorphic curves $\mathbf{H}^2/\Gamma(k)$ for various k using the functions from [**2**]. We use these to obtain partial information about gap sequences that we believe wasn't known before. In some cases we will also construct half-canonical classes, i.e., forms of weight 1 that correspond to half-canonical class.

The structure of the paper is as follows. In the first section we will briefly review the theory from [**2**] and we will prove an explicit transformation formula for the relevant functions. Then we will give a general way of constructing differential forms using this formula. In the second paragraph we will look at particular examples of k and will show what kind of information one can get about modular curves using the theorem in Section 1.

1. Preliminaries. We assume that the reader is familiar with the basic notion and structure of modular curves. Here we will review the basics of θ functions and main results of [**2**].

Definition. Given $\left[\begin{smallmatrix}\varepsilon\\ \varepsilon'\end{smallmatrix}\right] \in \mathbf{R}^2$, $\tau \in \mathbf{H}$, $z \in \mathbf{C}$, we define

$$\Theta\begin{bmatrix}\varepsilon\\ \varepsilon'\end{bmatrix}(z,\tau) = \sum_{n\in\mathbf{Z}} \exp 2\pi i\bigg\{\frac{1}{2}\left(n+\frac{\varepsilon}{2}\right)^2\tau + \left(n+\frac{\varepsilon}{2}\right)\left(z+\frac{\varepsilon'}{2}\right)\bigg\}.$$

The series are uniformly and absolutely convergent on compact subsets of $\mathbf{C}\times\mathbf{H}$. The main property we need is the transformation formula for θ functions.

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