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ON TORSION AND MIXED MINIMAL ABELIAN GROUPS

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ABSTRACT. An abelian group is said to be minimal if it is isomorphic to all its subgroups of finite index. We obtain a complete characterization of such groups in the torsion case; in the case of mixed groups of rank 1 we obtain a characterization for some large classes of such groups.

1. Introduction. There are many possible ways of considering the concept of minimality within a general categorical setting. Some approaches centered on categories of topological spaces have recently yielded interesting results (see, e.g., [**5**] and [**6**]). An intrinsic part of the difficulty in deciding on what approach to take appears to arise from the fact that the obvious orderings available are only reflexive and transitive and are not anti-symmetric. In recent work (see [**4**]) the authors and Wallutis have investigated categories of abelian groups from a similar standpoint to that applied by McMaster et al. in [**6**] and [**7**] for topological spaces. The present work presents a related but quite different approach. An abelian group G is said to be *minimal* if G is isomorphic to every subgroup H of finite index in G. Interestingly, an identical concept has been the subject of recent investigation in non-abelian group theory under the title "hc-groups"; the work of Robinson and Timm (see [**11**]) shows the connection between such groups in the finitely generated case and certain aspects of connectedness for manifolds.

The classification of all minimal abelian groups seems to be a difficult problem. However, by restricting our considerations to torsion groups and certain classes of mixed groups, it is possible to obtain fairly complete classifications in terms of Ulm invariants.

Our notation is standard and largely in accord with Fuchs [**2**] and [**3**], which contains all undefined terms used herein; an exception is that we

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