ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 2, Summer 2003

PARA-ORTHOGONAL POLYNOMIALS IN FREQUENCY ANALYSIS

LEYLA DARUIS, OLAV NJÅSTAD AND WALTER VAN ASSCHE

1. Introduction. By a trigonometric signal we mean an expression of the form

$$x(m) = \sum_{j=1}^{I} \left(\alpha_j e^{im\omega_j} + \alpha_{-j} e^{im\omega_{-j}}\right), \tag{1.1}$$

and we assume $\alpha_{-j} = \overline{\alpha_j}$, and $\omega_{-j} = -\omega_j \in (0, \pi)$ for $j = 1, 2, \dots, I$. The constants α_j represent *amplitudes*, the quantities ω_j are *frequencies*, and m is discrete time. The frequency analysis problem is to determine the numbers $\{\alpha_j, \omega_j : j = 1, 2, \dots, I\}$, and $n_0 = 2I$ when values $\{x(m) : m = 0, 1, \dots, N-1\}$ (observations) are known.

The Wiener-Levinson method, formulated in terms of Szegő polynomials, can briefly be described as follows (the original ideas of the method can be found in [**12, 20**]). An absolutely continuous measure ψ_N is defined on $[-\pi, \pi]$ (or on the unit circle $\mathbf{T}$ through the transformation $\theta \mapsto z = e^{i\theta}$) by the formula

$$\frac{d\psi_N}{d\theta} = \frac{1}{2\pi} \left| \sum_{m=0}^{N-1} x(m) e^{-im\theta} \right|^2. \tag{1.2}$$

Here N is an arbitrary natural number. The measure gives rise to a positive definite inner product which determines a sequence $\{\Phi_n(\psi_N, z) : n = 0, 1, 2 \dots\}$ of monic orthogonal polynomials (Szegő polynomials). All the zeros of $\Phi_n(\psi_N, z)$ lie in the open unit disk.

Let $\varphi_n(\psi_N, z)$ be the orthonormal polynomials (with positive leading coefficient κ_n^N) with respect to ψ_N. Then we have

$$\varphi_n(\psi_N, z) = \kappa_n^N \Phi_n(\psi_N, z), \tag{1.3}$$

Received by the editors on September 30, 2002.
The first author was partially supported by Laguna University under contract 1802010204 and by Ministerio de Ciencia y Technología del Gobierno Español under contract BF2001-3411.
The third author's research is supported by INTAS 00-272 and research grant G.0184.02 of FWO-Vlaanderen.