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MULTIPLICATIVE *SK* INVARIANTS ON $\mathbf{Z}_n$-MANIFOLDS WITH BOUNDARY

TAMIO HARA

ABSTRACT. Let $\mathbf{Z}_n$ be the cyclic group of order n. In this paper, we study a map T for $\mathbf{Z}_n$-manifolds with boundary which takes values in the ring $\mathbf{Z}$ and is additive with respect to the disjoint union of $\mathbf{Z}_n$-manifolds. We call T a $\mathbf{Z}_n$-SK invariant if it is invariant under $\mathbf{Z}_n$-cuttings and pastings. Then T induces an additive homomorphism $T : SK_*^{\mathbf{Z}_n}(pt, pt) \to \mathbf{Z}$, where $SK_*^{\mathbf{Z}_n}(pt, pt)$ is a cutting and pasting group (SK group) of all $\mathbf{Z}_n$-manifolds. First we obtain a basis of a free $\mathbf{Z}$-module $\mathcal{I}_*^{\mathbf{Z}_n}$ of all these invariants by using the Euler characteristic $\overline{\chi}$ of manifolds with boundary. As a result, we determine the class of all multiplicative invariants, which includes $\overline{\chi}^{\mathbf{Z}_s}$ (and $\chi^{\mathbf{Z}_s}$) in particular.

Introduction. Let G be a finite abelian group. Throughout this paper, by a G-manifold we mean an unoriented compact smooth manifold (which may have boundary) with smooth G-action. In [**2**] and [**3**], we have studied an equivariant cutting and pasting theory (SK theory) $SK_*^G(pt, pt)$ based on G-manifolds by using the notion of G-slice types. We now consider a map T for G-manifolds which takes values in the ring $\mathbf{Z}$ of rational integers and is additive with respect to the disjoint union of G-manifolds. We call T a G-SK invariant if it is invariant under G-cuttings and pastings. Furthermore, such T is said to be multiplicative if $T(M \times N) = T(M)T(N)$ for any G-manifolds M and N. Let $\overline{\chi}(M) = \chi(M) - \chi(\partial M)$ for a pair $(M, \partial M)$ of G-manifold and its boundary, where χ is the Euler characteristic. Then $\overline{\chi}^H$ and χ^H are multiplicative G-SK invariants for any subgroup H of G, where $\overline{\chi}^H(M) = \overline{\chi}(M^H)$, $\chi^H(M) = \chi(M^H)$ and $M^H = \{x \in M \mid hx = x \text{ for any } h \in H\}$.

The main object of this paper is to study such kind of invariants when G is the cyclic group $\mathbf{Z}_n$ of order n, $n \geq 1$. Here $\mathbf{Z}_1$ is the trivial group $\{1\}$.

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