ROCKY MOUNTAIN JOURNAL OF MATHEMATICS Volume 34, Number 3, Fall 2004

HURWITZ SPACES AND BRAID GROUP REPRESENTATIONS

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ABSTRACT. We give a new construction of a Hurwitz space, which is a moduli space of all branched covers of the Riemann sphere having a given combinatorial description. The action of the fundamental group of the Hurwitz space on the homology of the branched cover gives rise to a linear representation of a finite index subgroup of the spherical braid group, or of a finite extension of such a subgroup. We construct examples of each of these two cases. Using a result of Fried, we use these representations to extract information about the dimension of the image of the Hurwitz space in the genus g moduli space.

0. Introduction. In this paper we investigate certain moduli spaces, Hurwitz spaces, of branched covers of the Riemann sphere S^2 and representations of finite index subgroups of the spherical braid group which arise from these Hurwitz spaces. (By *spherical* braid group, we mean the group of braids in the two-sphere; we will refer to the more classical group of braids in the plane as the *planar* braid group.)

Hurwitz spaces play an important role in realizing groups as Galois groups, a role which has been explored primarily by Fried and Völklein in [4, 5, 7, 14, 15], etc. In these works they have given a couple of constructions of Hurwitz spaces, examined their algebraic structure, and explored their applications to the inverse Galois problem. In Section 1 of this paper we give an alternative construction of the Hurwitz spaces, exhibiting them as homogeneous spaces of Aut (S^2) , the group of orientation preserving homeomorphisms of S^2 . This point of view enables us to prove that the universal cover of a Hurwitz space is homotopy equivalent to S^3 (see the discussion just after Proposition 4), which is equivalent to showing that the Teichmuller space of a sphere

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²⁰⁰⁰ AMS Mathematics Subject Classification. 32G15, 20F36, 20C12.

Key words and phrases. Branched cover, Hurwitz space, braid group, moduli space.

Work partially supported by National Science Foundation grants No. 9401516 and No. 9622928.

Received by the editors on February 5, 2002, and in revised form on March 15, 2002.