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GENUS OF A CANTOR SET

MATJAŽ ŽELJKO

ABSTRACT. We define a genus of a Cantor set as the minimal number of the maximal number of handles over all possible defining sequences for it. The relationship between the local and the global genus is studied for genus 0 and 1. The criterion for estimating local genus is proved along with the example of a Cantor set having prescribed genus. It is shown that some condition similar to 1-ULC implies local genus equal to 0.

1. Introduction. We will consider Cantor sets embedded in three-dimensional Euclidean space $\mathbf{E}^3$. A defining sequence for a Cantor set $X \subset \mathbf{E}^3$ is a sequence (M_i) of compact 3-manifolds M_i with boundary such that each M_i consists of disjoint cubes with handles, $M_{i+1} \subset \operatorname{Int} M_i$ for each i and $X = \cap_i M_i$. We denote the set of all defining sequences for X by $\mathcal{D}(X)$.

Armentrout [**1**] proved that every Cantor set has a defining sequence. In fact every Cantor set has many nonequivalent, see [**7**] for definition, defining sequences and in general there is no canonical way to choose one. One approach is to compress unnecessary handles in the given defining sequence for a Cantor set. A class for which this process terminates is characterized by some property similar to 1-ULC, see [**10**] for details. But in general this process is infinite so the "incompressible" defining sequence may not exist. Hence we look at the minimal number of the maximal number of handles over all possible defining sequences for it and take the defining sequence for which this number is minimal. Unfortunately this sequence need not to be canonical, but the minimal number, i.e. *the genus*, itself has some interesting properties.

Using different terminology Babich [**2**] actually proved that the genus of a wild scrawny, see [**2**] for definition, Cantor set is at least 2.

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