

TYPE SUBMODULES AND DIRECT SUM DECOMPOSITIONS OF MODULES

JOHN DAUNS AND YIQIANG ZHOU

ABSTRACT. A type decomposition of a module M over a ring R is a direct sum decomposition for which any two distinct summands have no nonzero isomorphic submodules. In this paper, we investigate when a module possesses certain kinds of type decompositions and when such decompositions are unique.

Introduction. It is well known that every torsion abelian group has a unique decomposition into its p -torsion subgroups. By Goodearl-Boyle [4], every nonsingular injective module E has a unique decomposition $E = E_1 \oplus E_2 \oplus E_3$ where E_1, E_2, E_3 are of types I, II, III respectively, see Definition 2.7. Why do such decompositions exist? Why are such decompositions unique? Are there any common things between these two results? All these questions will be answered in this paper. In fact, we can present a more general theory on existence and uniqueness of type decompositions of modules, so that the above results, as well as many other known results, are obtained as very special cases.

The common property for certain diverse kinds of direct sum decompositions of modules M including the two decompositions above is that any two distinct direct summands have no nonzero isomorphic submodules, or equivalently all direct summands are what we will call type submodules. The cause for the existence of such decompositions is that these modules M have a ‘decomposability property’ which will be discussed in detail in Section 1, while the uniqueness of such direct sum decompositions is ensured by a module property called UTC. A theory of such modules is developed in Section 2.

Throughout, all rings R are associative with identity and modules are unital right R -modules and M is an R -module. A class $\mathcal{K}$ of modules is a *type*, or *natural class*, if it is closed under isomorphic copies,

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