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SYMPLECTIC GEOMETRY OF VECTOR BUNDLE MAPS OF TANGENT BUNDLES

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ABSTRACT. If (M, g) is a Riemannian manifold, then TM has a canonical almost Kähler structure. The derivative of a map of Riemannian manifolds rarely preserves the Kähler forms of the tangent bundles, even up to conformality. Thus we define a weakening of symplectomorphism, called H-isotropic map and study the H-isotropy of vector bundle maps.

1. Introduction and notation. If L is a submanifold of an almost Hermitian manifold (N, J, g, ω), $\omega = g(J\cdot, \cdot)$, then the normal bundle $L^\perp$ of L also possesses an almost Hermitian structure $(\hat{J}, \hat{g}, \hat{\omega})$. Here $\hat{\omega}$ is called the canonical almost symplectic structure of $L^\perp$ (cf. [**4**]). An interesting problem in symplectic geometry is: when are ω and $\hat{\omega}$ isomorphic? (Cf. [**6**], [**4**].) A job relevant to this problem is to study vector bundle maps between two such bundles $L_1^\perp$ and $L_2^\perp$ (e.g., [**4**, Theorem 4.1]). The tangent bundle of a Riemannian manifold can be thought of as a special case of a normal bundle of an almost Hermitian manifold [**4**]. Moreover, the almost symplectic form on TM is in fact just a pull-back of the canonical symplectic form on T^*M. Thus we are motivated to study the symplectic geometry of vector bundle maps of tangent bundles of Riemannian manifolds.

Suppose (M, g) is a Riemannian manifold. Then TM is equipped with Sasaki metric $\hat{g}$ [**8**], [**2**]. If $X \in \Gamma(TM)$, then we use X^H and X^V to denote its horizontal and vertical lifts to TM, respectively. An almost complex structure J for TM compatible with $\hat{g}$ is defined as follows: $J(X_\xi^H + Y_\xi^V) = X_\xi^V - Y_\xi^H$ [**2**]. The 2-form $\omega := \hat{g}(J\cdot, \cdot)$ is exactly $D^*(\omega_c)$ where $D : TM \to T^*M$ is the dual map induced by g and ω_c is the canonical symplectic form on T^*M [**2**]. Thus we call $(J, \hat{g}, \omega)$ the *canonical almost Kähler structure* of TM. While $\hat{g}$ has been studied extensively, little seems to have been done about ω.

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