JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 19, Number 4, Winter 2007

ATTRACTION PROPERTIES OF UNBOUNDED SOLUTIONS FOR A NONLINEAR ABEL INTEGRAL EQUATION

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ABSTRACT. In this paper a nonlinear Abel integral equation with a power nonlinearity is considered. This equation has a solution which is unbounded in zero, i.e., is unbounded on $[0, \delta)$, for every positive δ. Attraction properties for this solution are studied. We show that it is always possible to find functions attracted by the unbounded solution, as well as functions not attracted by such a solution.

1. Introduction. In this paper we consider the Abel integral equation

$$u(x) = \int_0^x (x-s)^\alpha u(s)^\beta \, ds, \tag{1}$$

where $x \geq 0$ and $(\alpha, \beta) \in (-1, 0) \times (-1/\alpha, +\infty)$. This equation is a particular case of a nonlinear Volterra integral equation with convolution kernel

$$u(x) = \int_0^x k(x-s) g(u(s)) \, ds; \tag{2}$$

where the kernel k and the nonlinearity g satisfy the following properties:

• k is a positive function of $L^1_{\text{loc}}(\mathbf{R}^+)$, such that $K(x) = \int_0^x k(s)\, ds$ is a strictly increasing function.

• g is a continuous strictly increasing function, such that $g(0) = 0$, $g' > 0$ almost everywhere, and transforms null sets into null sets.

Keywords and phrases. Nonlinear Abel integral equations, attracting behavior, attraction basins.
This paper has been partially supported by Project MTM2004-06226 of CICYT.
Received by the editors on March 20, 2006, and in revised form on November 20, 2006.

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