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COUNTEREXAMPLES FOR ABSTRACT LINEAR VOLTERRA EQUATIONS

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ABSTRACT. The following objects exist:

1) An abstract Volterra equation in a Banach space with an exponentially bounded scalar kernel, such that the resolvent exists but is not exponentially bounded.

2) An analytic resolvent operator corresponding to a self adjoint negative definite operator in a Hilbert space, and a scalar kernel a, such that $1/|\hat{a}|$ grows faster than polynomially.

3) An analytic function F defined on the right half plane and satisfying $|F(s)| \leq M/|s|$ on the right half plane, such that F is not the Laplace transform of an L^∞ function on the positive half axis.

1. Introduction. This paper deals with the resolvent operator of an abstract Volterra integral equation

$$u(t) = \int_0^t a(t-s)Au(s)\,ds + f(t). \tag{1.1}$$

Here A is an unbounded linear operator in some Banach space X, f is an X-valued function, and a is a scalar valued function. By a *resolvent operator* we mean a strongly continuous family $\{S(t) : t \geq 0\}$ of bounded linear operators in X satisfying

$$S(t)Ax = AS(t)x \quad \text{for all } x \in \operatorname{dom}(A),\ t \geq 0;$$
$$S(t)x = x + A\int_0^t a(t-s)S(s)x\,ds \quad \text{for all } x \in X,\ t \geq 0.$$

The solution to (1.1) is then given—at least formally—by

$$u(t) = \frac{d}{dt}\int_0^t S(t-s)f(s)\,ds.$$

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