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ORTHOGONAL POLYNOMIALS FOR THE SOLUTION OF SEMI LINEAR TWO-POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT. We present an integral equation method for the solution of a class of nonlinear two-point boundary value problems. The method relies on the use of the Kumar-Sloan transformation and uses special orthogonal polynomials to efficiently implement a Galerkin method for the solution of the resulting nonlinear integral equation. Numerical examples show the rapid convergence for smooth solutions which is a consequence of approximation theorems of Jackson.

1. Introduction. In this article we present a numerical method for the approximate solution of the semi linear two-point boundary value problem

$$\begin{aligned} \left(\frac{d}{dx}\right)^2 u(x) &= f(x, u(x)), \quad x \in [0,1], \\ u(0) &= u(1) = 0. \end{aligned} \tag{1.1}$$

When $f(x,u) = f(x)$, we refer to this problem as the Poisson equation on $[0,1]$. Instead of the homogeneous boundary conditions, we might also consider nonhomogeneous conditions, but then we can transform the problem to an equation of the form (1.1) with a new nonlinearity. In this paper we will assume that f is sufficiently regular and fulfills certain growth conditions to guarantee that (1.1) has solutions, see [**1, 7, 10**].

We use the Kumar-Sloan transformation, see [**1, 3, 5**], to solve (1.1) and therefore we investigate the numerical solution of the nonlinear integral equation

$$v(x) = f\left(x, \int_0^1 G(x,y)\, v(y)\, dy\right), \quad x \in [0,1], \tag{1.2}$$

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