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COMPUTING POSITIVE FIXED-POINTS OF DECREASING HAMMERSTEIN OPERATORS BY RELAXED ITERATIONS

ALVISE SOMMARIVA AND MARCO VIANELLO

ABSTRACT. We prove global convergence of (under)relaxed Picard-like methods for fixed-point equations $u = A(u), A : C_+(\Omega) \to C_+(\Omega)$, Ω being a compact Hausdorff space. The operator A is *decreasing* and completely continuous, and possesses no pairs of distinct and comparable coupled-fixed points. Infinite- as well as finite-dimensional Hammerstein equations of this type arise in transport theory. As a numerical application, we test Picard, updated Picard, Jacobi, and Gauss-Seidel (under)relaxed iterations on the discrete "decreasing" version of Chandrasekhar H-equation. A comparison with popular Newton-like solvers is also presented.

1. Introduction. In this paper we consider as a model problem the Hammerstein equation

$$u(x) = A(u)(x) = KN(u)(x), \quad x \in \Omega, \tag{1}$$

where $K : C_+(\Omega) \to C_+(\Omega)$ is (the restriction of) a linear completely continuous operator, $C(\Omega)$ denoting the space of continuous real functions on the compact Hausdorff space Ω (endowed with $\|\cdot\|_\infty$), and $C_+(\Omega)$ its positive cone; cf. [**16, 18**]. In (1), N is the Nemytskii operator

$$N(u)(x) = f(x, u(x)), \quad x \in \Omega, \tag{2}$$

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