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SOLUTIONS AND MULTIPLE SOLUTIONS FOR SECOND ORDER PERIODIC SYSTEMS WITH A NONSMOOTH POTENTIAL

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ABSTRACT. A nonautonomous second order system with a nonsmooth potential is studied. Using the nonsmooth critical point theory, first an existence theorem is proved. Then, by strengthening the hypotheses on the nonsmooth potential, a multiplicity theorem is proved using the nonsmooth second deformation. The hypotheses on the nonsmooth potential make the Euler functional of the problem bounded below but do not make it coercive. Moreover, the analytical framework of the paper incorporates strongly resonant periodic systems.

1. Introduction. In this paper, we consider the following second order periodic system with a nonsmooth potential:

$$(1)\qquad \begin{cases} -x''(t) - A(t)x(t) \in \partial j(t, x(t)) & \text{a.e. on T=[0,b]}, \\ x(0) = x(b), \ x'(0) = x'(b). \end{cases}$$

Here $t \to A(t)$ is a continuous map from $T = [0, b]$ with $b > 0$ into the space of $N \times N$-symmetric matrices and $j(t, x)$ is a measurable function defined on $T \times \mathbf{R}^N$, which is locally Lipschitz and in general nonsmooth in the $x \in \mathbf{R}^N$ variable. By $\partial j(t, x)$ we denote the generalized (Clarke) subdifferential of $x \to j(t, x)$ (see Section 2).

Rabinowitz [**17**] examined problem (1) under the assumptions that, for every $t \in T$, $A(t)$ is strictly negative definite, $j \in C^1(T \times \mathbf{R}^N)$ and $j(t, \cdot)$ exhibits a strictly superquadratic growth. He proved an existence result using the saddle point theorem. Note that the hypothesis that $A(t)$ is strictly negative definite implies that the spectral decomposition of the nonlinear differential operator has trivial negative and zero parts. Mawhin [**12**] (see also Mawhin and Willem [**13**]) assumed that, for all

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