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REAL ZEROS OF THREE DIFFERENT CASES OF POLYNOMIALS WITH RANDOM COEFFICIENTS

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ABSTRACT. There are many known asymptotic estimates for the expected number of real zeros of a trigonometric polynomial $V(\theta) = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + \cdots + a_n \cos n\theta$ with independent identically distributed random coefficients. However, recent study of random matrix theory as well as self-reciprocal random *algebraic* polynomials has led to development of a different class of random trigonometric polynomials. These applications, as well as, of course, mathematical interests, motivate us in this paper to provide asymptotic estimates for the expected number of zeros and the level-crossings of three different, albeit closely related, random trigonometric polynomials. The first two forms of random trigonometric polynomials have the self-reciprocal property of $a_j = a_{n-j}$. The third case is a random trigonometric polynomial which is in exact form arising from a transformation of an algebraic polynomial with the above self-reciprocal properties. It is shown that the expected number of real zeros of the classical trigonometric polynomial has been reduced by half, while for the other two cases this expected number remains the same as the previous cases.

1. Introduction. In this paper we study three different cases of random trigonometric polynomials. The first case is of classical form $V(\theta) = \sum_{j=0}^{n} a_j \cos j\theta$, in which the jth coefficient is equal to the $(n-j)$th term, that is, $a_j = a_{n-j}$. As we noted in Farahmand [**6**], this assumption on the coefficient naturally arises in the study of self-reciprocal random algebraic polynomials, see also [**7**]. Therefore, it is of interest to number theorists as well as, of course, those interested in the mathematical behavior of random polynomials, to study $V(\theta)$. The second case is the random trigonometric polynomial $R(\theta) = \sum_{j=0}^{n}(a_j \cos j\theta + b_j \sin j\theta)$, where $a_j = a_{n-j}$ and $b_j = b_{n-j}$. $T(\theta) = \sum_{j=0}^{n-1}\{\alpha_{n-j}\cos(j+1/2)\theta + \beta_{n-j}\sin(j+1/2)\theta\}$ is the third case we study, which is produced as seen in [**6**] from a self-reciprocal random algebraic polynomial. These three cases have some similarity on means

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