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NOTES ON NEW (ANTISYMMETRIZED) ALGEBRAS

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ABSTRACT. We define the simple non-associative algebra $N(e^{A_S}, q, n, t)_k$ and its simple subalgebras in this work. We also prove that the anti-symmetrized algebra $N(e^{A_S}, q, n, t)^-_{[k]}$ is simple. There are various papers on finding all the derivations of an associative algebra, a Lie algebra and a non-associative algebra (see [**3, 5–7, 9, 12, 14–16**]). We also find all the derivations $\mathrm{Der}_{\mathrm{anti}}(N(e^{\pm x^r}, 0, 0, 1)^-_{[2+]})$ of the anti-symmetrized algebra $N(e^{\pm x^r}, 0, 0, 1)^-_{[2+]}$, and every derivation of the algebra is outer in this paper.

1. Preliminaries. Let $\mathbf{N}$ be the set of all non-negative integers and $\mathbf{Z}$ the set of all integers. Let $\mathbf{N}^+$ be the set of all positive integers. Let $\mathbf{F}$ be a field of characteristic zero and $\mathbf{F}^\bullet$ the set of all non-zero elements in $\mathbf{F}$. For fixed integers $i_1, \dots, i_m$, we define S_m as the set $\{x_1^{i_1} \cdots x_m^{i_m}, x_1^{i_1} \cdots x_{m-1}^{i_{m-1}}, \dots, x_2^{i_2} \cdots x_m^{i_m}, \dots, x_1^{i_1}, \dots, x_m^{i_m}\}$. Throughout the paper, n and t are given non-negative integers, and m denotes a non-negative integer such that $m \le n + t$. For any subset S of S_m and $q \le n$, we can define the $\mathbf{F}$-algebra $\mathbf{F}[e^{\pm[S]}, q, n, t] := \mathbf{F}[e^{\pm[S]}, \ln(x_1)^{\pm 1}, \dots, \ln(x_q)^{\pm 1}, x_1^{\pm 1}, \dots, x_n^{\pm 1}, x_{n+1}, \dots, x_{n+t}]$ spanned by

$$\mathbf{B} = \{e^{a_1 s_1} \cdots e^{a_r s_r} \ln(x_1)^{d_1} \cdots \ln(x_q)^{d_q} x_1^{j_1} \cdots x_{n+t}^{j_{n+t}} | s_1, \dots, s_r \in S,$$
$$a_1, \dots, a_r, d_1, \dots, d_q \in \mathbf{Z},\ j_1, \dots, j_n \in \mathbf{Z}, j_{n+1}, \dots, j_{n+t} \in \mathbf{N}\}$$

where, throughout the paper, we put $\ln(x_u)^{d_u} := (\ln(x_u))^{d_u}$, $1 \le u \le q$. Note that, if $t \ge 1$, then $\mathbf{F}[e^{\pm[S]}, q, n, t]$ is a semi-group ring not a group ring (see [**17**]). We then denote $\partial_{h_1}^{p_1} \cdots \partial_{h_r}^{p_r}$ as the composition of the partial derivatives $\partial_{h_1}, \dots, \partial_{h_r}$ on $\mathbf{F}[e^{\pm[S]}, q, n, t]$ and ∂_h^0, $1 \le h \le n+t$, denotes the identity map on $\mathbf{F}[e^{\pm[S]}, q, n, t]$ where $0 \le h_1, \dots, h_r \le$

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