

FILTERS IN ORDERED Γ -SEMIGROUPS

KOSTAQ HILA

ABSTRACT. In this paper we characterize the principal filters on any ordered Γ -semigroup M and their structure and properties are investigated by using the relation \mathcal{N} which is the smallest complete semilattice congruence on M . In particular, we prove that every principal filter of any ordered Γ -semigroup M can be uniquely determined by its \mathcal{N} -classes of M . Also, by using the relation \mathcal{N} , we will observe that \mathcal{N} on any ordered Γ -semigroup M is the equality relation on M if and only if M is a semilattice such that $a \leq a\gamma a$ for all $a \in M$, $\gamma \in \Gamma$, and \mathcal{N} is the universal relation on M if and only if M is the only principal filter. We also investigate properties of the complete semilattice congruence classes of M .

1. Introduction and preliminaries. In 1987, Kehayopulu [8] introduced the concept of filter in poe-semigroups. Later Kehayopulu [12] defined the relation \mathcal{N} on a po-semigroup and obtained some results. Various kinds of ordered semigroups have been widely studied by many authors [1, 2, 8–14, 17, 20] by using the notion of filter and the relation \mathcal{N} . In [15] Kwon introduced the concept of filter and the relation \mathcal{N} in ordered Γ -semigroups and obtained some results extending those for ordered semigroups. Also, in [3, 4] we have used these notions to characterize some classes of ordered Γ -semigroups. In the present paper we give some new results extending those for ordered semigroups, dealing with the principal filters on any ordered Γ -semigroup M and their structure and properties, which are investigated by using the relation \mathcal{N} which is the smallest complete semilattice congruence on M . In particular, we prove that every principal filter of any ordered Γ -semigroup M can be uniquely determined by its \mathcal{N} -classes of M . Also, we will consider a structure of principal filter on ordered Γ -semigroups and by using the relation \mathcal{N} , we will observe that \mathcal{N} on any ordered Γ -semigroup M is the equality relation on M if and only if M is a semilattice having the property $a \leq a\gamma a$ for all $a \in M$,

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