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A GENERAL FIXED POINT THEOREM FOR MULTI-VALUED MAPPING IN UNIFORM SPACE

D. TÜRKOĞLU AND B.E. RHOADES

ABSTRACT. We establish a general fixed point principle which includes known fixed point theorems in uniform spaces. Then examples show that this theorem includes known fixed point theorems and also yields a new theorem.

1. Introduction. A fixed point theorem for multi-valued contraction mappings was proved for the first time by Nadler [**15**]. Since then, many authors have given generalizations of this theorem in various forms, such as the one given by Wegrzyk. Wegrzyk has applied fixed point theorems to the proof of multi-valued functions and functional equations [**25**].

Uniform spaces form a natural extension of metric spaces. An exact analogue of the well-known Banach contraction principle in uniform spaces was obtained independently by Acharya [**1**] and Taraftar [**21**]. Since then a number of fixed point theorems for single-valued and multi-valued mappings using various contractive conditions in this setting have been obtained ([**2, 6–10, 12–14, 18–20**]). In this paper we first prove a fixed point theorem for a multi-valued map in hyperspace. Then examples show that this theorem includes known fixed point theorems and yields a new theorem.

Let $(X, \mathcal{U})$ be a uniform space. A family $P = \{d_i : i \in I\}$ of pseudometrics on X with indexing set I, is called an associated family for the uniformity $\mathcal{U}$ if the family

$$\beta = \{V(i,r) : i \in: I, r > 0\}$$

where

$$V(i,r) = \{(x,y) : x, y \in X, d_i(x,y) < r\}$$

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