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NUMERICAL INDEX OF BANACH SPACES OF WEAKLY OR WEAKLY-STAR CONTINUOUS FUNCTIONS

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ABSTRACT. We show that the space of weakly continuous functions from a compact Hausdorff space into a Banach space has the same numerical index as the range space. We also establish some inequalities for the numerical index of the space of weakly-star continuous functions from a compact Hausdorff space into the dual of a Banach space.

The numerical index of a Banach space is a constant relating the norm and the numerical radius of operators on the space. Let us present the relevant definitions. For a Banach space X, we write B_X for the closed unit ball, S_X for the unit sphere, X^* for the dual space and $\Pi(X)$ for the subset of $X \times X^*$ given by

$$\Pi(X) = \{(x, x^*) \in X \times X^* : x^*(x) = \|x^*\| = \|x\| = 1\}.$$

For a bounded linear operator T on X, we define its *numerical radius* by

$$v(T) = \sup\{|x^*(Tx)| : (x, x^*) \in \Pi(X)\}.$$

The *numerical index* of the space X is then given by

$$\begin{aligned} n(X) &= \max\{k \geq 0 : k\,\|T\| \leq v(T) \quad \text{for all} \quad T \in L(X)\} \\ &= \inf\{v(T) : T \in L(X),\ \|T\| = 1\}, \end{aligned}$$

where $L(X)$ stands for the space of all bounded linear operators on X.

The concept of numerical index was first suggested by Lumer in 1968. At that time, it was known that a Hilbert space of dimension

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