ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 3, Summer 1989

THETA SERIES OF DEGREE 2 OF QUATERNARY QUADRATIC FORMS

R. SCHULZE-PILLOT

Introduction . The purpose of this note is to investigate linear combinations of theta series of degree two of quaternary positive definite integral quadratic forms from a representation theoretic point of view, using the expression of theta series with the help of the oscillator or Weil representation of the metaplectic group. In the case that the quadratic form is the norm form of a definite quaternion algebra we find that certain linear combinations of the theta series of the forms in a given (similitude) genus are related to the Maass spezialschar (also called Saito-Kurokawa space). Such a connection did (in special cases) already appear in the work of Yoshida [**15, 16**]. Using Andrianov's zeta functions [**1, 2**] one can use these results to deduce relations between representation numbers. As always, the representation theoretic formulation of the results generalizes beyond the original setting and allows number fields and indefinite forms whenever the involved integrals converge.

I wish to thank S. Rallis, who helped me understand the representation theoretic situation considered here. I also wish to thank the University of Maryland for its hospitality.

1. Basics. Let V be a vector space over $\mathbf{Q}$ of dimension m with positive definite quadratic form q and bilinear form $B(x,y) = q(x+y) - q(x) - q(y)$. The group of proper similitudes of (V,q) will be denoted by $\mathrm{GSO}(V)$. For a lattice L on V ($q(L) \subseteq \mathbf{Z}$) the theta series of degree n is

$$\theta(L,z) = \sum_{x \in L^n} \exp(2\pi i \,\mathrm{tr}\,(q(x)Z))$$

(where $Z \in \mathcal{H}_n$, the Siegel upper half space of genus n, $x = (x_1, \ldots, x_n)$ and $q(x) = {}^1\!/_2(B(x_i,x_j))$ is the matrix of scalar products of $x_1, \ldots, x_n$ with respect to B).

Work done at the University of Maryland, College Park, and supported by a Forschungsstipendium of the Deutsche Forschungsgemeinschaft.