REGULARITY AND UNIQUENESS OF CERTAIN SYSTEMS OF FUNCTIONS ANNIHILATED BY A FORMALLY INTEGRABLE SYSTEM OF VECTOR FIELDS

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1. Introduction and the statement of the main theorems. It is well known that a local CR diffeomorphism of a smooth CR manifold of CR codimension 1 with a nondegenerate Levi form is determined by a finite number of constants. Moreover, if M and M' are real analytic (C^∞, respectively) CR manifolds as above and $F : M \to M'$ is a CR diffeomorphism of class C^7 then F is real analytic (C^∞, respectively). These are consequences of the existence of the invariant Cartan connection on the bundle of pseudo conformal frames over M ([3], cf. also [9]). If M' is a real hypersurface in $\mathbf{C}^{n+1}$ the above two facts are easier to see: Let r be a local defining function of M' and let $\{L_1, \ldots, L_n\}$ be an independent set of C^ω tangential Cauchy-Riemann vector fields on M. Then the components of $F = (f_1, \ldots, f_{n+1})$ satisfy an equation

$$(1) \qquad\qquad r \cdot F = 0$$

and a system of partial differential equations

$$(2) \qquad L_i f_j = 0, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n+1.$$

Through a process of repeated differentiation of (1), reduction of order of derivatives using (2) and introducing new variables, we can construct a C^ω pfaffian system whose integral manifolds correspond to CR diffeomorphisms of M onto M'. The regularity and the uniqueness of F follow from the Frobenius theorem (cf. [6]). This method is a variant of the so-called 'prolangation' originated by E. Cartan, which he used as a basic tool for the equivalence problem (cf. [2]).

The purpose of this paper is to generalize the above properties of CR diffeomorphisms to certain systems of functions annihilated by a

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