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NECESSARY AND SUFFICIENT CONDITIONS FOR MULTIPARAMETER BIFURCATION

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ABSTRACT. Obstruction theory is used in order to give a complete characterization of linearized local and global bifurcation. In both cases there is a set of two topological invariants, depending only on the linear part, such that, if both are trivial, there is a nonlinear part with no local or global bifurcation. The nonvanishing of any of these invariants is sufficient for bifurcation for any nonlinearity.

0. Introduction. A bifurcation problem is the study of the zeros of the nonlinear map $f(\lambda, x)$, where λ belongs to the parameter space Λ, x to a space E and $f(\lambda, x)$ has values in another space F near a known family of solutions $(\lambda, x(\lambda))$ called the trivial solutions. After linearization, one may assume that $x(\lambda) \equiv 0$ and that $f(\lambda, x)$ has the form

$$(A_0 - A(\lambda))x - g(\lambda, x) \tag{1}$$

where $A(0) = 0, g(\lambda, x) = o(\| x \|)$. It is well known that a necessary condition for bifurcation is that A is non-invertible and if A is a Fredholm operator one may write (1), near $(0,0)$, as

$$\begin{aligned}&(A_0 - QA(\lambda))(x_2 - (I - KQA(\lambda))^{-1}KQ(A(\lambda)x_1 + g(\lambda, x))\\&\ominus (I - Q)(A(\lambda)((I - KQA(\lambda))^{-1}x_1\\&+ x_2 - (I - KQA(\lambda))^{-1}KQ(A(\lambda)x_1 + g(\lambda, x))\\&+ (I - A(\lambda)KQ)^{-1}g(\lambda, x)),\end{aligned}$$

where $x = x_1 \oplus x_2, x_1$ in $\ker A_0$, Q is a projection on Range A_0 and K is the pseudo-inverse of A_0 from Range A_0 into X_2, the complement

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