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ORLICZ SPACES WHICH ARE RIESZ ISOMORPHIC TO ℓ^∞

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ABSTRACT. The main purpose of this paper is to describe, in terms of the function φ and the measure μ, Orlicz spaces $L^\varphi(S,\sum,\mu)$ which are Riesz isomorphic to ℓ^∞. The "thickness", in the sense of Baire category, of the subset of measures for which $L^\varphi(S,\sum,\mu)$ is Riesz isomorphic to ℓ^∞ is also investigated.

I. Basic notation and auxiliary results. Throughout the note, in what concerns Riesz spaces (= vector lattices) we use the terminology of [**2**]. When two Riesz spaces L and K are Riesz isomorphic, then this fact will be noted by $L \simeq K$. The symbols $\mathbf{R}^S$ and N are reserved for the space of functions from a set S into $\mathbf{R}$ with the standard pointwise order and for the set of positive integers, respectively. Moreover, e_s denotes the characteristic function of the set $\{s\}, L_+$ is the cone of positive elements of a Riesz space L and $\ell_0^\infty(S)$ is the ideal in $\ell^\infty(S)$ consisting of functions with at most countable support. When S is countable then, of course, $\ell_0^\infty(S) = \ell^\infty(S)$.

We start with two simple lemmas.

LEMMA 1. *Let $L_i (i = 1,2)$ be Riesz subspaces of $\mathbf{R}^S$ containing all e_s's. If $T : L_1 \to L_2$ is a Riesz isomorphism onto, then there exists a function $g \in \mathbf{R}_+^S$ and a bijection $\alpha : S \to S$ such that*

$$T(x)(s) = g(s)x(\alpha(s))$$

for all $x \in L_1$.

The above statement follows immediately from two facts: $T(e_s)$ is an atom in L_2 (so it has the form $a_s e_{s'}$) and T is a normal Riesz homomorphism.

The next Lemma will be frequently used.

LEMMA 2. *Let L be a Riesz subspace of $\mathbf{R}^S$ containing all e_s's, and let A be a subset of S. If L is Riesz isomorphic to $\ell_0^\infty(S)$, then*

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