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REFLEXIVE ALGEBRAS OF MATRICES

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ABSTRACT. We study some sufficient properties for an algebra of matrices to be reflexive. In particular we show that a semismple algebra is reflexive. Commutative algebras are then considered, and it is seen that a commutative algebra of 3×3 matrices is reflexive if either it can be diagonalized or it is of dimension 2. Finally we show that the algebra of all operators which leave invariant every element of a complemented lattice of subspaces forms a semisimple algebra. This is related to a result by Harrison and Longstaff on reflexive lattices of subspaces.

1. Introduction. Let V be a vector space of finite dimension n over the complex number $\mathbf{C}$. The algebra of all linear operators an V is denoted by Hom V. The algebra of $n \times n$ matrices over $\mathbf{C}$ is denoted by M_n.

The set of all subspaces of V is a modular lattice under the operations intersection (meet) and sum (join) of two subspaces. Further, any sublattice of this lattice is again modular.

Let $\mathscr{L}$ be a lattice of subspaces of V and $\mathscr{A}$ a subalgebra of Hom (V). We define the operations Alg and Lat as follows. Alg $\mathscr{L}$ is the set (necessarily an algebra) of all $A \in$ Hom V which leave invariant every subspace $W \in \mathscr{L}$. Similarly Lat $\mathscr{A}$ is the lattice of all subsapaces of V which are left invariant by every element of $\mathscr{A}$. $\mathscr{L}$ (respectively $\mathscr{A}$) is called reflexive iff Lat Alg $\mathscr{L} = \mathscr{L}$, (Alg Lat $\mathscr{A} = \mathscr{A}$ respectively). The classification of reflexive algebras and reflexive lattices is far from complete even in finite dimensional spaces, although some progress has been made (cf. **[1, 3, 5, 6, 11]**). It is worth noting, however, that every finite dimensional algebra is isomorphic to a reflexive one (cf. Brenner and Bulter, J. London Math. Soc. **40** (1965), 183–187). In this paper we shall study reflexivity and give a more algebraic proof of a result due to Harrison and Longstaff **[7]**. We shall also study some particular types of algebras such as commutative algebras of matrices. We close with a discussion of subspaces lattices which may be useful in generating examples.

In what follows all lattices will contain $\{0\}$ and V, and all algebras will contain the identity, I, except for certain subalgebras of nilpotent matrices.

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