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SPACES FORMED BY SPECIAL ATOMS, I

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"Dedicated to Ismenia Sales de Souza, my wife"

1. Introduction. C. Fefferman and E. M. Stein [3] and R. R. Coifman [1] observed that a real-valued function f in $L_1(T)$, (where T is the perimeter of the unit disk in the complex plane) is the real part of a boundary function $F \in H^1(\mathbf{D})$ $(F \in H^1(\mathbf{D})$ if and only if $\|F\|_{H^1} = \operatorname{Sup}_{r<1} \int_T |F(re^{i\theta})|\, d\theta < \infty$, where $\mathbf{D} = \{z \in C;\ |z| < 1\})$ if and only if there is a sequence (a_n), of atoms and a sequence (c_n), of numbers, such that $\sum_{n=1}^{\infty} |c_n| < \infty$ and $f(t) = \sum_{n=1}^{\infty} c_n a_n(t)$. (A real valued function defined on T is called an *atom* whenever a is supported on an interval $I \subset T$, $|a(t)| \leqq |I|^{-1}$ and $\int_I a(t)dt = 0$.) Moreover, letting $\lambda(f)$ equal the infimum of $\sum_{n=1}^{\infty} |c_n|$ over all such representations of f, there exist absolute constants M and N such that $M\|F\|_{H^1} \leqq \lambda(f) \leqq N\|F\|_{H^1}$; we shall denote the set of all such f as ReH^1 and $\|f\|_{ReH^1} = \lambda(f)$.

ReH^1 is well known as the atomic decomposition of $H^1(\mathbf{D})$. C. Fefferman and E. M. Stein in their famous paper [3], proved that the dual of ReH^1 is the space $BMO = \{f \in L_1(T);\ \|f\|_{BMO} = \operatorname{Sup}_{I \subset T}(1/|I|) \int_I |f(t) - f_I| dt < \infty\}$ where $f_I = (1/|I|) \int_I f(t)dt$, originally introduced by F. John and L. Nirenberg [4]. BMO stands for bounded mean oscillation and I is an interval.

In this paper we introduce a new function space B defined by $B = \{f\colon T \to \mathbf{R}, f(t) = \sum_{n=1}^{\infty} c_n b_n(t);\ \sum_{n=1}^{\infty} |c_n| < \infty\}$. Each b_n is a special atom, that is, a real-valued function b, defined on T, which is either $b(t) \equiv 1/2\pi$ or $b(t) = -(1/|I|)\chi_R(t) + (1/|I|)\chi_L(t)$, where I is an interval on T, L is the left half of I and R is the right half. $|I|$ denotes the length of I and χ_E the characteristic function of E. B is endowed with the norm $\|f\|_B = \operatorname{Inf} \sum_{n=1}^{\infty} |c_n|$, where the infimum is taken over all representations of f, which becomes a Banach space. At this point, a natural question is: Is B topologically equivalent to ReH^1? In other words, do there exist positive constants C and D such that $C\|f\|_B \leqq \lambda(f) \leqq D\|f\|_B$? Regarding

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