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ASYMPTOTIC STABILITY OF A COUPLED DIFFUSION SYSTEM ARISING FROM GAS-LIQUID REACTIONS

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ABSTRACT. This paper is concerned with the asymptotic behavior of the time dependent solution in relation to the corresponding steady-state solution for a nonlinear coupled reaction-diffusion system arising from gas-liquid absorption. Existence and uniqueness of both time-dependent and steady-state solutions are discussed, and various boundary conditions are included in the discussion. It is shown in the case of a homogeneous system that for any non-negative initial function the time dependent solution converges exponentially to zero as $t \to \infty$ when the boundary condition is of either Dirichlet or mixed type. However, for Neumann type boundary condition, multiple constant steady-state solutions exist and the time-dependent solution may converge to any one of these steady-states. Depending on the relative magnitude between the initial functions, convergence of the time-dependent solution to one of these constant states is explicitly given. For a nonhomogeneous system with nonzero boundary or internal data the convergence of the time-dependent solutions also depends on the relative magnitude between the components of the steady-state solution. A characterization of the stability and instability of a steady-state solution is established, and in the case of stability an estimate of the stability region is given.

1. Introduction. In the theory of a gas-liquid diffusion reaction system in a p-dimensional medium Ω the concentration of the dissolved gas $u = u(t, x)$ and the reactant $v = v(t, x)$ are governed by the coupled reaction-diffusion equations (cf. [**2**–**4**, **6**, **12**])

$$\begin{aligned} u_t - D_1 \Delta u &= -k_1 uv \\ v_t - D_2 \Delta v &= -k_2 uv \end{aligned} \qquad (t > 0,\ x \in \Omega) \tag{1.0}$$

where Δ is the Laplacian operator, D_1, D_2 are the diffusion coefficients, k_1, k_2 are the reaction rate constants and $r_i = -k_i uv$ represent the rate of reactions. A more general reaction rate is given by

$$r_1(u, v) = -k_1 u^m v^{m'}, \quad r_2(u, v) = -k_2 u^n v^{n'}$$

and is called the (m, n)th order reaction (cf. [**4**]). Motivated by the above

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