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THE REDUCED THEORY OF QUADRATIC FORMS

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Two central problems in the theory of quadratic forms over fields are the computation of the set of equivalence classes of quadratic forms over a field (which reduces to the computation of the Witt ring) and the computation of the set of values taken on by a given quadratic form (or equivalently, the determination of when a given quadratic form represents zero nontrivially). Recently, a "reduced theory" of quadratic forms has provided a partial solution to these problems by the computation of the Witt ring modulo its nil radical and the computation of the additive semigroup generated by the value set of a quadratic form. Our intention here is to provide an efficient and fairly self-contained exposition of these results to the reader knowing the rudiments of the algebraic theory of quadratic forms (mainly, the definition of the Witt ring) and Pfister's local-global principle. These prerequisites can all be found in a few chapters of either of the books of Lam, Scharlau or Milnor-Husemoller [**7**, **14**, **10**]. The necessary valuation theory can be found, for example, in Ribenboim's book [**13**].

Some of the results and many of the arguments here are new. Little use is made of the formalism of residue class forms and none of semiorderings. We do emphasize real-valued places (which allow applications of the Stone-Weierstrass theorem as well as valuation theory) and closely examine the maximal preorders over which a form is anisotropic. In spite of innovations, however, our main goals are expository and we have borrowed on accasion from the arguments as well as the results of others (especially including Becker and Bröcker [**1**]).

In §1 we introduce the notions of equivalence and isotropy of forms with respect to a preorder. The questions of isotropy and representability with respect to an arbitrary preorder are reduced in §2 to preorders consistent with only finitely many real-valued places. The main theorems on the structure of the reduced Witt ring and on representability are in §§3 and

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