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THE MEHLER-FOCK TRANSFORM OF DISTRIBUTIONS

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1. **Introduction.** The classical Mehler-Fock transformaton of a function $f(x)$ is defined by

$$f^*(\tau) = \int_0^\infty f(x)P_{-(1/2)+i\tau}(\text{ch}x)\text{sh}x\, dx$$

where $P_{-(1/2)+i\tau}(\text{ch}x)$ is the Legendre function of first kind. Although the corresponding inversion formula

$$f(x) = \int_0^\infty \tau\text{th}\pi\tau P_{-(1/2)+i\tau}(\text{ch}x)f^*(\tau)\, d\tau$$

was given by Mehler [7] in the year 1881 in a purely formal way, the research on this transformation has been rather slow due to the complex nature of this transformation. Fock [3] in the early forties established these formulas for a class of functions. Since the early sixties considerable interest has been shown in the use of this transformation in solving the boundary value problems in the mathematical theory of elasticity. The objective here is to extend this transformation to a class of distributions.

The notation and terminology used here is that of Zemanian [15]. In the following I denotes the open interval $(0, \infty)$. Spaces $\mathscr{D}(I)$ and $\mathscr{D}'(I)$ have their usual meaning.

The Legendre function $P_{-(1/2)+i\tau}(\text{ch}x)$ possesses the following well known properties [3, p. 254].

$$\left| P_{-(1/2)+i\tau}(\text{ch}x) \right| < \frac{x}{2\text{Sh}(x/2)} \leqq 1, \qquad x \geqq 0, \tag{1}$$

$$P_{-(1/2)+i\tau}(\text{ch}\theta) = \left(\frac{\theta}{\text{Sh}\theta} \right)^{1/2} \left\{ J_0(\tau\theta) + \frac{1}{8\tau}\left(\coth\theta - \frac{1}{\theta}\right) J_1(\tau\theta) + \cdots \right\}, \tag{2}$$

where $J_n(\tau\theta)$ is the Bessel function of first kind [3, p. 253].

Using the asymptotic expansions of Bessel functions for large $\tau > 0$ and fixed $\theta > 0$ in (2) it follows that

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