ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 9, Number 4, Fall 1979

FUZZY TOPOLOGIES CHARACTERIZED BY NEIGHBORHOOD SYSTEMS

RICHARD H. WARREN

In this paper we prove that neighborhood systems are an equivalent method for determining fuzzy topologies. This characterization of fuzzy topology uses the definition of neighborhood of a point which was given in [3] and used in [4] to describe continuity between fuzzy topological spaces. The sequence of development in this paper parallels Chapter 9 in [2].

In [1] the authors give a different definition for the neighborhood of a point and then are able to characterize a proper subclass of all fuzzy topologies on a fixed set. The difficulty with their definition is that distinct fuzzy topologies can have the same system of neighborhoods.

DEFINITION 1. Let X be a set. A *fuzzy set* in X is a function from X into $[0, 1]$, the closed unit interval. So g is a fuzzy set in X iff $g : X \to [0, 1]$. To each set $E \subset X$ corresponds the "crisp" fuzzy set μ_E which is the characteristic function of E.

We shall continue the pattern begun in [4] of relating fuzzy sets to other fuzzy sets by the function operations of $=$, $\leqq$, $<$, $\wedge$, $\vee$, $\bigvee$, $\bigwedge$, $+$ and $-$, where $f \leqq g$ iff $f(x) \leqq g(x)$ for all $x \in X$ and $f + g$ is defined by $(f + g)(x) = f(x) + g(x)$, with similar definitions for the other operations in the list. Negation of each of the operations is determined by one $x \in X$. It is assumed that a supremum (infimum) of fuzzy sets taken over an empty index set is μ_ϕ (μ_X).

DEFINITION 2. Let X be a set and let T be a family of fuzzy sets in X. Then T is called a *fuzzy topology* on X iff it satisfies the conditions:

(a) 0 $(= \mu_\phi)$ and 1 $(= \mu_X)$ are in T;

(b) if $g_i \in T$, $i \in I$, then $\bigvee \{g_i : i \in I\} \in T$;

(c) if $g, h \in T$, then $g \wedge h \in T$.

The pair (X, T) is called a *fuzzy topological space* (abbreviated as fts). The elements of T are called *open* fuzzy sets. By a fuzzy set in a fts (X, T), we mean a fuzzy set in X.

DEFINITION 3. [3]. A fuzzy set n in a fts (X, T) is a *neighborhood of a point* $x \in X$ iff there exists $g \in T$ such that $g \leqq n$ and $n(x) = g(x) > 0$.

Received by the editors on June 21, 1977.