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FIXED POINTS OF f-CONTRACTIVE MAPS

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1. **Introduction.** Let (X, d) be a metric space. A fixed point of a map $g: X \rightarrow X$ is a common fixed point of g and the identity map 1_X of X. Motivated by this fact, we replace 1_X by a continuous map $f: X \rightarrow X$, and obtain the following.

DEFINITIONS. Let f be a continuous self-map of X. Then a self-map g of X is said to be *f-contractive* if $d(gx, gy) < d(fx, fy)$ for all $x, y \in X$, $gx \neq gy$.

Let C_f denote the family of all maps $g: X \rightarrow X$ such that $gX \subset fX$ and $gf = fg$. Given a point $x_0 \in X$ and a map $g \in C_f$, *an f-iteration of x_0 under g* is a sequence $\{fx_n\}_{n=1}^{\infty}$ such that $fx_n = gx_{n-1}$.

We observe that an f-contractive map is always continuous. Note that given $x_0 \in X$, its f-iteration under g is not unique; however, in case $f = 1_X$, these definitions reduce to the usual ones.

We give conditions under which f-contractive maps have fixed points. In fact, necessary and sufficient conditions for the existence of fixed points of continuous self-maps of X are given. In order to do this, criteria for an f-iteration to be Cauchy are of interest. In this direction, Geraghty [5] obtained important results on usual contractive maps and iterations.

In this paper, we generalize results of Edelstein [4], Rakotch [7], and Geraghty [5] on the existence of fixed points, and, consequently, obtain many extended forms of the Banach contraction principle, especially those of Boyd-Wong [2], [8], Geraghty [5], Jungck [6], and Rakotch [7].

In § 2, basic n.a.s.c.'s for the existence of fixed points of self-maps of an arbitrary metric space and their applications are given.

In § 3, we give a n.a.s.c. that an f-iteration of $x_0 \in X$ under g be convergent. This condition is used to prove criteria for the existence of fixed points for metric spaces more general than complete ones. Some applications are also considered.

Throughout this paper, X denotes a metric space with metric d, and f denotes always a continuous self-map of X.

2. **General existence theorems.** In this section, we give some n.a.s.c.'s for the existence of fixed points of a continuous self-map f of X. First, we need the following.

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