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SINGULAR BOUNDARY PROBLEMS FOR THE DIFFERENTIAL EQUATION $Lu = \lambda\sigma u$

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1. **Introduction.** The classical theory of Sturm-Liouville boundary problems for second-order differential operators on finite intervals has served as the point of departure for a number of modern generalizations. An extensive literature exists in connection with equations of the type $Lu = \lambda u$, where L is a linear differential operator of order $n \geqq 1$ on a finite or infinite interval I, and λ is a complex parameter. Boundary conditions imposed upon solutions of these equations lead to differential boundary problems which are termed "singular" if I is infinite, or if the coefficients of the differential operator have a singular behavior near the endpoints of a finite interval. One method of dealing with such problems, due originally to H. Weyl, has been used effectively for a wider class of problems, notably by N. Levinson, E. A. Coddington, and F. Brauer. It consists of the replacement of the given problem by a sequence of regular (i.e. nonsingular) problems on finite subintervals which tend to the original interval. Known results for these regular problems then yield information about the singular case through a limiting process. This procedure may be carried out even though the so-called "singular" problem is not explicitly defined at the outset; the results obtained in the limit as the finite subintervals tend to the original interval are then defined as constituting the solution of a singular problem associated with the differential operator in question. The merit of this approach is that it does not require one to know in advance what boundary conditions, if any, are appropriate for the direct definition of a singular problem. However, such direct definition has been given by M. H. Stone, E. A. Coddington, and others, for important cases involving formally self-adjoint L with associated operators which are symmetric or selfadjoint in the Hilbert space of functions square-integrable in the interval I.

A further generalization of problems of this type leads to the consideration of

$$Lu = \lambda Mu, \tag{i}$$

where L and M are differential operators defined on I. Just as in the

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