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COMPARISON THEOREMS FOR SECOND ORDER DELAY DIFFERENTIAL EQUATIONS[1]

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1. **Introduction.** Consider the delay differential equations

$$x''(t) + M(t)x(t) + N(t)x(t - \Delta(t)) = 0\,, \tag{1.1}$$

$$x''(t) + m(t)x(t) + n(t)x(t - \Delta(t)) = 0\,, \tag{1.2}$$

where $M(t)$, $N(t)$, $m(t)$, $n(t)$, and $\Delta(t) \geqq 0$ are defined and continuous on $[0, B)$, $B \leqq +\infty$. In case $n \equiv 0 \equiv N$, the classical Sturm comparison theorem says that whenever $M(t) \geqq m(t)$ every solution of (1.1) must have a zero between consecutive zeros of a nontrivial solution of (1.2). The equation

$$x''(t) + \tfrac{1}{2}x(t) - \tfrac{1}{2}x(t - \pi) = 0 \tag{1.3}$$

shows that such a theorem is in general not possible for equations (1.1), and (1.2), for (1.3) has both oscillatory and nonoscillatory solutions, viz. $x(t) = \sin t$ and $x(t) \equiv 1$.

Initial value problems for delay equations of the above type are posed in the following way (see [**6**, Chapter I, §2]). Given a function $\varphi(t)$ continuous on the initial set

$$I = \{t - \Delta(t) : t - \Delta(t) \leqq 0\} \cup \{0\}$$

and given a real number r, one seeks a solution $x(t)$ such that

$$x(t - \Delta(t)) \equiv \varphi(t - \Delta(t)), \qquad t - \Delta(t) \in I, \tag{1.4}$$

$$x'(0+) = r. \tag{1.5}$$

In this paper we obtain results which allow us to estimate the first zero of a solution of (1.1) corresponding to initial functions which do not change sign on the initial set in terms of the first positive zero of nontrivial solutions of (1.2) corresponding to the identically zero initial function. As important applications of these results we obtain uniqueness theorems for solutions of boundary value problems (BVP's) for nonlinear delay differential equations.

The following example serves to illustrate our results.

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