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THE COMPUTATION OF THE SPECTRA OF HIGHLY OSCILLATORY FREDHOLM INTEGRAL OPERATORS

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ABSTRACT. We are concerned with the computation of spectra of highly oscillatory Fredholm problems, in particular with the *Fox-Li* operator

$$\int_{-1}^{1} f(x)\mathrm{e}^{\mathrm{i}\omega(x-y)^2}\mathrm{d}x = \lambda f(y), \quad -1 \le y \le 1,$$

where $\omega \gg 1$. Our main tool is the finite section method: an eigenfunction is expanded in an orthonormal basis of the underlying space, resulting in an algebraic eigenvalue problem. We consider two competing bases: a basis of Legendre polynomials and a basis consisting of modified Fourier functions (cosines and shifted sines), and derive detailed asymptotic estimates of the rate of decay of the coefficients.

Although the Legendre basis enjoys in principle much faster convergence, this does not lead to much smaller matrices. Since the computation of Legendre coefficients is expensive, while modified Fourier coefficients can be computed efficiently with FFT, we deduce that modified Fourier expansions, implemented in a manner that takes advantage of their structure, present a considerably more effective tool for the computation of highly oscillatory Fredholm spectra.

1. Introduction. Our understanding of highly oscillatory phenomena and their computation has advanced in leaps and bounds in the last decade. In particular, the subject matter of highly oscillatory quadrature is, to all intents and purposes, satisfactorily understood, and there exists a wealth of efficient and affordable numerical methods for integrals of the form

$$\int_{\Omega} f(x)\mathrm{e}^{\mathrm{i}\omega g(x)}\mathrm{d}x,$$

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