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ON EXISTENCE AND UNIQUENESS OF THE MILD SOLUTION FOR FRACTIONAL SEMILINEAR INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we consider the existence and uniqueness of the mild solution for the fractional integro-differential equation

$$\frac{d^a x(t)}{dt^a} = Ax(t) + g(t, x(t)) + \int_{t_0}^{t} f(t, s, x(s))\, ds,$$

where $0 < a \le 1$, g and f are given functions.

1. Introduction. Let d^α/dt^α denote the Caputo fractional derivative of order α, for $0 < \alpha \le 1$. We consider the following integro-differential equation

$$(1) \qquad \begin{cases} \dfrac{d^\alpha x(t)}{dt^\alpha} = Ax(t) + g(t, x(t)) \\ \qquad\qquad + \displaystyle\int_{t_0}^{t} f(t, s, x(s))\, ds \quad t > t_0 \ge 0, \\ x(t_0) = x_0 \in X \end{cases}$$

where A is a generator of a strongly continuous semigroup $\{T(t) : t \ge 0\}$ on the Banach space X, $f : D \times X \to X$ and $g : I_h \times X \to X$ is continuous in t, for

$$I_h := [t_0, t_0 + h] \quad \text{and} \quad D := \{(t, s) : t_0 \le s \le t \le t_0 + h\}, \quad h > 0.$$

Using a fixed point theorem, we prove the existence and uniqueness of a mild solution for equation (1). The nonlinearities $g(t, x(t))$ and $f(t, s, x(s))$ are assumed to satisfy some conditions, given later.

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