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UNIQUE SOLVABILITY OF NUMERICAL METHODS FOR STIFF DELAY-INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. This paper deals with the unique solvability of numerical methods for stiff delay-integro-differential equations (DIDEs). Several unique solvability conditions of the extended general linear methods for DIDEs are derived. The conclusions obtained are applied to some common numerical methods such as the extended linear multistep methods and the extended Runge-Kutta methods. In the end, concrete examples illustrate the utility of the theory.

1. Introduction. Delay-integro-differential equations (DIDEs) arise widely in the mathematical modelings of physical and biological phenomena. Significant advances in the research of theoretical solutions and numerical solutions for such equations have been made in recent years (see, e.g., [**8, 9, 10**]). A survey of the related results refers to Brunner's monograph (cf. [**1**]). The existing research deals mainly with stability, dissipativity, convergence and computational implementation of the numerical methods. When numerically computing a stiff DIDE, generally speaking, an implicit algebraic equation needs to be solved. In order to obtain highly effective numerical methods, the concept of *algebraic stability* is often used. The algebraic stability, unfortunately, cannot guarantee the existence of numerical solutions. For example, Crouzeix, Hundsdorffer and Spijker [**3**] constructed a counterexample, which shows that the fourth order Runge-Kutta method

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