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SOLVABLE INFINITE FILIFORM LIE ALGEBRAS

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ABSTRACT. An infinite filiform Lie algebra L is residually nilpotent and it is graded associated with respect to the lower central series, has smallest possible dimension in each degree, but is still infinite. This means that $\mathrm{gr}(L)$ is of dimension two in degree one and of dimension one in all higher degrees. We prove that if L is solvable, then already $[L, L]$ is abelian. The isomorphism classes in this case are given in [**1**], but the proof is incomplete. We make the necessary additional computations and restate the result in [**1**] when the ground field is the complex numbers.

1. Introduction. Infinite filiform Lie algebras have been studied among others by Fialowski [**2**], Millionshchikov [**4**] and Shalev-Zelmanov [**5**]. They may be seen as projective limits of finite dimensional filiform Lie algebras introduced by Vergne [**6**] as nilpotent Lie algebras with maximal degree of nilpotency among all nilpotent Lie algebras of a certain dimension. One result is that there is only one infinite filiform naturally graded Lie algebra L, where naturally graded means that L is isomorphic to its graded associated with respect to the filtration defined by the lower central series ($L^1 = L,\ L^{i+1} = [L, L^i]$, $i \geq 1$). This Lie algebra, denoted M_0, has a basis $a, e_1, e_2, \ldots$ with $[a, e_i] = e_{i+1}$ and $[e_i, e_j] = 0$ for all i and j. We have that M_0 is generated by a, e_1 which is a basis for the component of degree 1, and e_i is a basis for the component of degree i for $i \geq 2$. A general infinite filiform Lie algebra L may be seen as a (filtered) deformation of M_0 such that $\mathrm{gr}(L) \cong M_0$. Thus we have the following definition:

Definition 1.1. An infinite filiform Lie algebra is a Lie algebra L with a basis $a, e_1, e_2, \ldots$ satisfying

$$[a, e_j] = e_{j+1} \quad \text{and} \quad [e_j, e_{j+1}] = \sum_{s=1}^{\infty} \lambda_{js} e_{2j+1+s} \quad \text{for all } j \geq 1$$

for some λ_{js}, $j, s \geq 1$ such that for each j, $\lambda_{js} = 0$ for $s \gg 1$.

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