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DIFFERENTIABILITY WITH RESPECT TO INITIAL FUNCTIONS FOR PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH UNBOUNDED DELAY

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ABSTRACT. Initial value problems for first order partial functional almost linear equations with unbounded delay are considered. The functional dependence is represented by the generalized Hale operator with the values in the abstract normed space. The suitable system of axioms for the phase space is given.

A theorem on the global existence of classical solutions and continuous dependence upon initial data is formulated. The proof is based on the method of successive approximations. Finally, a result on the differentiability of solutions with respect to initial functions is proved. Important examples of integral differential equations or differential functional equations with a deviated argument can be obtain by specializing given functions.

1. Introduction. Write $B = (-\infty, 0] \times [-b, b]$, $b \in R^n_+$, $R_+ = [0, +\infty)$. Vectorial inequalities are understood to hold componentwise. For $t \in [0, a]$ where $a > 0$, we put $E_t = (-\infty, t] \times R^n$. Suppose that $z : E_a \to R^k$ and $(t, x) \in E_a$. We consider the function $z_{(t,x)} : B \to R^k$, defined by

$$z_{(t,x)}(s, y) = z(t + s, x + y), \quad (s, y) \in B. \tag{1}$$

Let X be a linear space with the norm $\| \cdot \|_X$ consisting of functions mapping the set B into R^k. We denote by $M_{k\times n}$ the set of all $k \times n$ matrices with real elements. Put $E = [0, a] \times R^n$, and assume that

$$f : E \to M_{k\times n}, \qquad f = [f_{ij}]_{i=1,\dots,k,j=1,\dots,n},$$
$$F : E \times X \to R^k, \qquad F = (F_1, \dots, F_k),$$

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