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Some Theorems on Bounded Analytic Functions.

By

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Recently various theorems on bounded analytic functions have been generalized to the case of a multiply connected domain by several authors. For instance, the well-known Schwarz's lemma has been generalized by Ahlfors, Garabedian and Nehari. The theory of complete orthonormal system of analytic functions and of Szegö kernel function play important rôles in such generalizations. The objectives of this paper are to generalize Hardy's theorem on bounded functions (sec. 1–3) and to give a radius of univalence for a certain class of bounded functions (sec. 4–6).

1. Let D be a finite domain in the complex z-plane, bounded by n closed analytic curves Γ_i, $(i=1, 2,\dots, n)$. We define the class Λ of all functions $f(z)$ satisfying the following conditions:

(i) $f(z)$ is single-valued and regular at $z \in D$,

(ii) $f(z)$ has an integral $F(z)=\int_{z_0}^{z} f(t)\,dt$ $(z_0 \in D)$ which may have additive moduli if z describes a closed contour in D but which is continuous in the closed domain $\bar{D}=D+\Gamma$ $(\Gamma=\sum_{i=1}^{n}\Gamma_i)$,

(iii) for every function $f(z)$ there corresponds a summable function $\mu(t)$ on Γ with its square of absolute value $|\mu|^2$, such that for any two points t_1 and t_2 on the same boundary curve Γ_i $(i=1, \dots, n)$

$$F(t_2)-F(t_1)=\int_{t_1}^{t_2}\mu(t)dt$$

holds for the right determinations of the multivalued function $F(z)$. $\mu(t)$ is called the associate function on Γ of the function $f(z)$ in D. It is evident that if $f(z)$ is regular in D and continuous in $\bar{D}$, it belongs to the class Λ and its associate function $\mu(t)$ is given by the boundary value $f(t)$ of $f(z)$.

We consider the space Λ and introduce in Λ the metric