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Note on linear processes

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§1. Introduction

As is well known, K. Karhunen [5] introduced the canonical representation of stationary processes which plays an important role in the theory of linear prediction. Let $X(t)$, $-\infty<t<+\infty$, be a mean continuous, purely non-deterministic weakly stationary process with $EX(t)\equiv 0$. Then it can be expressed in the form

$$X(t)=\int_{-\infty}^{t} F(t-u)\,dZ(u) \tag{1}$$

with an orthogonal random measure dZ such that $E(dZ(u))^2=du$. This representation is not unique, but there exists essentially one and only one representation which satisfies the condition

$$\mathfrak{M}_t(X)=\mathfrak{M}_t(Z)\,, \qquad \text{for every } t, \tag{2}$$

where $\mathfrak{M}_t(X)$ and $\mathfrak{M}_t(Z)$ are the closed linear manifolds spanned by $X(\tau)$, $\tau\leq t$, and $Z(\tau)-Z(\sigma)$, $\tau, \sigma\leq t$, respectively. Such a representation (1) is called the canonical representation of $X(t)$ and F is called the canonical kernel. Using the canonical representation, the linear predictor $U(s, t)$ of $X(t)$ based on $X(\tau)$, $\tau\leq s$ $(\leq t)$, i.e. the projection of $X(t)$ into $\mathfrak{M}_t(X)$ can be expressed in the form

$$U(s, t)=\int_{-\infty}^{s} F(t-u)\,dZ(u)\,.$$

One of us introduced the multiple Markov property for Gaussian processes [3]. This concept can be defined for purely non-deterministic stationary processes which are not always Gaussian, by replacing the independence with the orthogonality. Let $X(t)$ be a