J. Math. Kyoto Univ.
3-1 (1963) 115-125.

Unique factorization of ideals in the sense of quasi-equality

To Professor Y. Akizuki for celebration of his 60th birthday

By

Toshio NISHIMURA

(Communicated by Prof. Nagata, Aug. 20, 1963)

Introduction

Throughout this paper, let R be an integral domain, i.e., a commutative ring with an identity and having no proper zero-divisors, and let K be the field of quotients of R. By an R-module, we shall mean in this paper an R-module contained in K. Let A and B be R-modules, then the set of all elements x in K such that xb is in A for every element b of B is denoted by A/B. In the special case that $A=R$, R/B is often denoted by B^{-1}, and we write $(B^{-1})^n$ by B^{-n}, for brevity. By an ideal of R, we mean a non-zero *fractional* ideal of R. If $A\subseteqq R$, then we say that A is an *integral* ideal of R. If $(A^{-1})^{-1}=A$, then we say that A is a *V-ideal* of R. If $(A^{-1})^{-1}=A$ and $AA^{-1}=A$, then we say that A is an *F-ideal* of R. It is known (cf. Mori [1]) that an F-ideal is an integral ideal and is characterized by the properties that (1) A^{-1} is a ring containing R and (2) A is a V-ideal. If $A^{-1}=B^{-1}$, then we say that A is *quasi-equal to* B and write $A\sim B$.

In this paper we shall prove the following theorem.

Theorem. *The following three conditions are equivalent to each other:*

(1) *Any ideal A of R satisfies a quasi-equality of the following type:*

$$A \sim \mathfrak{p}_1^{r_1}\mathfrak{p}_2^{r_2}\cdots\mathfrak{p}_n^{r_n},$$

where $\mathfrak{p}_i$ $(i=1, 2, \cdots, n)$ are prime ideals in R and r_i $(i=1, 2, \cdots, n)$