J. Math. Kyoto Univ.
3-2 (1964) 207-216.

The expected number of zeros of continuous stationary Gaussian processes*

Dedicated to Professor Y. Akizuki for his sixtieth birthday

By

Kiyosi ITO

(Received Jan. 16, 1964)

1. Introduction

Let $x(t)=x(t, \omega)$, $\omega \in \Omega(\mathcal{B}, P)$ be a stationary Gaussian process with continuous sample paths. Then the mean $a=E[x(t)]$ is independent of t and the covariance function $r(t)=E[(x(s+t)-a)(x(s)-a)]$ is an even function of t, independent of s, expressible in the form

$$r(t) = \int_{-\infty}^{\infty} e^{i\lambda t} dF(\lambda) \tag{1}$$

with a bounded measure dF symmetric with respect to 0.

Let $N=N(\omega)$ be the number of zeros of the sample path of $x(t)$ in $0<t<T$ and $N_c=N_c(\omega)$ the number of crossings of the level c by the sample path of $x(t, \omega)$.

The purpose of this paper is to prove

Theorem.

$$E(N) = E(N_c) = \frac{T}{\pi}\sqrt{-\frac{r''(0)}{r(0)}} \exp\left(-\frac{a^2}{2r(0)}\right) \tag{2}$$

where $r''(0)$ is the second Schwarz derivative, i.e.,

* This work was supported in part by the National Science Foundation, Grant 16319, and in part by the Office of Naval Research, Contract Nonr 225 (28) at Stanford University.