

Remarks on generalized rings of quotients

II

By

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Introduction. In a recent paper of the writer we defined the notion of generalized rings of quotients; they are certain rings contained in the total quotient ring of the original ring. But rings of quotients are not necessarily subrings of the total quotient ring. Therefore in §1 of this paper, we generalize the notion of generalized rings of quotients so that we cover completely the rings of quotients and previous generalization of rings of quotients. Thus those we shall treat are rings which are contained in the total quotient ring of a homomorphic image of a ring having similar properties as previously defined generalized rings of quotients. We prove that the kernel of such a homomorphism is also the kernel of a ring of quotients (in the usual sense). In §2 we shall prove a theorem on the weak global dimension of a ring which is a slight generalization of a result contained in [6]. Rings are always commutative rings with units.

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§1. First, we give a few results on flatness which are well-known.

Lemma 1. *Let R, R' be rings and let $f: R \rightarrow R'$ be a homomorphism. Then R' is flat as an R -module if and only if for every maximal ideal $\mathfrak{m}'$ of R' , $R'_{\mathfrak{m}'}$ is $R_{\mathfrak{m}}$ -flat with $\mathfrak{m} = f^{-1}(\mathfrak{m}')$.*