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On some uniqueness questions in primary representations of ideals

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0. Introduction

In a noetherian ring, every ideal can be represented as an irredundant intersection of finitely many primary ideals. There are several uniqueness properties associated with such a representation, for example see [11; Chap. IV, §5, Theorems 6, 7, 8]. The major part of this paper is devoted to constructing examples to show that these uniqueness properties do not hold when an ideal is an infinite irredundant intersection of primary ideals.

We begin with a discussion of the notion of associated prime divisor of an ideal. We consider four definitions of associated prime divisor which appear in the literature, and show that that of Nagata [8; p. 19] is the most general. However the Zariski-Samuel characterization, that P is an associated prime divisor of the ideal A if $A:(x)$ is P-primary for some x, is the one which is relevant when we study irredundant primary representations.

In §2 we study irredundancy in the representation of the radical $\sqrt{A}$ of the ideal A as the intersection of its minimal prime divisors. We find that, if A has infinitely many minimal prime divisors, these

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