

## Classical flows with discrete spectra

Dedicated to Professor Atuo Komatsu for his 60th birthday

By

Toshio NIWA

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### Introduction

In his paper [12] von Neumann showed that unitary equivalence of flows implies their metrical equivalence in the case of ergodic flows with discrete spectra. More precisely, if two one-parameter groups of unitary operators induced by two ergodic flows have discrete spectra and are unitarily equivalent, then these flows are metrically equivalent. Moreover they can be realized "canonically" as rotations on compact abelian groups.

Up to the present time many results are obtained with regard to the set of eigenvalues, which forms an additive subgroup of real numbers, and eigenfunctions. However this does not finish the investigation of the eigenvalues, eigenfunctions, discrete spectrum, and etc., if we consider the flows  $(\varphi_t)$  on the manifolds  $M$  generated by the differential equations on them. For instance, we do not know even whether the ranks of the additive groups of eigenvalues of  $(\varphi_t)$  are finite or not. [1], [2].

In this paper we consider the case when flow  $(\varphi_t)$  is ergodic and the manifold  $M$  is compact, then we can consider  $M$  as the total space of a locally trivial smooth fibre space, whose base space is a torus and fibres are submanifolds of  $M$ , and moreover  $\varphi_t$  is