Some results on PL-cobordism

By

Franklin P. PETERSON[1]

Dedicated to Prof. Atuo Komatu on his sixtieth birthday

(Communicated by Professor H. Toda, November 9, 1968)

1. Introduction

In this paper we will use the results of Sullivan [10], Stasheff [9], and Peterson and Toda [8] concerning the classifying spaces $BSPL$ and BSF to obtain some further results concerning $H^*(BSPL)$[2] and the p-torsion in Ω^{PL}_*, the oriented PL-cobordism ring. Since some of these results are computational in nature, we will often only give an outline of the proof.

Let $J_{PL}: BSPL \to BSF$ be the natural map. Let $q_i \in H^{ir}(BSF)$ be the Wu class ($r = 2p - 2$ throughout). Our first main result is that $J^*_{PL}(\beta q_i) \neq 0$ if $i \geq p + 1$. (It is easy to show that $J^*_{PL}(\beta q_i) = 0$ if $i \leq p$). Our other main result is a computation of the p-torsion of Ω^{PL}_* in dimensions $\leq p^2 r$. In particular, we show that there is a PL-manifold M of dimension $(2p+1)r - 1$ of order p^2 in Ω^{PL}_* such that pM is not detected by any ordinary characteristic numbers. Finally, we make a few conjectures concerning $H^*(MSPL)$.

2. $H^*(BSPL)$

Sullivan [10] has proved that $BSPL$ is of the same mod p homotopy type as $BSO \times B\mathrm{Coker}\,J$, where B Coker J is a space

---

1) Partially supported by a Fulbright Fellowship.
2) All cohomology groups in this paper have coefficients Z_p, p an odd prime, unless otherwise stated.