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Flatness of an extension of a commutative ring

Dedicated Professor K. Asano for his sixtieth birthday

By

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Throughout the present paper, we mean by a ring a commutative ring with identity and by a module a unitary one. Let R be a ring and let A be a homomorphic image of the polynomial ring $R[X]$ of a set of variables X with kernel I. The main purpose of the present paper is to discuss some topics related to the following

Theorem 1. *Assume that I is the principal ideal generated by $f(X)=a_0X^{(0)}+a_1X^{(1)}+\cdots+a_nX^{(n)}$ ($a_i \in R$; $X^{(i)}$ monomials, $X^{(i)} \neq X^{(j)}$ if $i \neq j$). Let J be the ideal $\sum a_iR$ generated by the coefficients a_i of $f(X)$. Then A is R-flat if and only if J is a direct summand of R (i.e., $J=eR$ with an element $e \in R$ such that $e^2=e$).*

1. Preliminary results.

Besides very well known elementary facts on flatness, we use the following two results:

Lemma 1.1. *Assume that R and R^* are noetherian rings such that R^* is an R-module. Let ϕ be the homomorphism from R into R^* such that $\phi a = a \cdot 1$ (in R^*). Let $\mathfrak{M}^*$ be the set of maximal ideals of R^* and let $\mathfrak{M}$ be the set of prime ideals $\mathfrak{m}$ of R such that $\mathfrak{m}=\phi^{-1}(\mathfrak{m}^*)$ with $\mathfrak{m}^* \in \mathfrak{M}^*$. Then R^* is a flat R-module if*