J. Math. Kyoto Univ. (JMKYAZ)
11-3 (1971) 449-484

First order hyperbolic mixed problems

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(Received April 5, 1971)

§1. Introduction

We consider the mixed problems for the first order hyperbolic systems in a quarter space, $t>0$, $x>0$, $y\in\mathbf{R}^{n-1}$;

$$(1.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}u(t)=Lu(t)+f(t) \\ u(0)=g \\ Bu(t)|_{x=0}=h, \end{cases}$$

where $L=A\dfrac{\partial}{\partial x}+\sum_{j=1}^{n-1}B_j\dfrac{\partial}{\partial y_j}+K$, A, B_j and K are $N\times N$ matrices and B is a $l\times N$ matrix.

The aim of this article is to derive energy inequalities of the solutions for the mixed problems (1.1).

We assume as follows;

A. I) The coefficients of (L, B) are independent of t, sufficiently smooth with respect to (x, y) in $\mathbf{R}^n$ and constant outside a compact set in $\mathbf{R}^n$. The coefficients of L are real valued and A is non singular.

A. II) is strictly hyperbolic, that is, $A\xi+\Sigma B_j\eta_j$ has only real distinct eigen values for $(x, y)\in\mathbf{R}^n$, $(\xi, \eta)\in\mathbf{R}^n$, $(\xi, \eta)\neq 0$. Hence

$$M(x, y; \lambda, \eta)=A^{-1}(\lambda-i\Sigma B_j\eta_j),\ \mathrm{Re}\,\lambda>0,\ \eta\in\mathbf{R}^{n-1}$$

has not real eigen values. Let k of eigen values have negative real parts. Then we can find a smooth $N\times N$ matrix $U(x, y: \lambda, \eta)$ homo-