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Projective modules over polynomial rings over division rings

By

S. PARIMALA and R. SRIDHARAN[1)]

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Introduction. It was proved in [3] that if D is any (non-commutative) division ring, then there exist non-free projective ideals in $D[X, Y]$. The aim of this paper is to study the set of isomorphism classes of finitely generated projective modules over $D[X, Y]$, where D is a division algebra which is finite-dimensional over its centre. In §1. we prove a proposition on projective modules over matrix rings and deduce (Cor. 1. 3) that if D is a finite-dimensional central division algebra of dimension n^2 over K and L a splitting field for D, then for any finitely generated projective module P over $D[X, Y]$, $L\otimes_K P$ is free over $M_n(L)[X, Y]$. If we choose a splitting field L for D which is a finite Galois extension of K with Galois group G and an isomorphism $L\otimes_K D[X, Y] \xrightarrow{\sim} M_n(L)[X, Y]$, we get a cocycle $f: G \longrightarrow \mathrm{Aut}_{L[X,Y]-alg} M_n(L)[X, Y]$. For any integer $m \geqslant 1$, let $Z^1(m)$ denote the set of maps $T: G \longrightarrow \mathrm{Aut}_{L[X,Y]} M_n(L)[X, Y]^m$, where T satisfies a suitable cocycle condition and $T(\sigma)$ is $f(\sigma)$-semilinear for every $\sigma \in G$. We prove (Th. 2. 1) in §2, that for $m \geqslant 1$, the set of isomorphism classes of finitelygenerated projective modules of rank m (where rank is defined in a suitable manner) is in bijection with a quotient set $H^1(m)$ of $Z^1(m)$ modulo an equivalence relation. In § 3, we show (Cor. 3. 2) that the

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