

Asymptotic behavior of multitype Galton-Watson processes

By

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0. Introduction

The asymptotic behavior of the distributions of multitype Galton-Watson processes has been studied by many mathematicians. According to the author's knowledge, Jirina [8] for subcritical processes is the first paper on this subject, and Chistyakov [4] and Mullikin [10] for critical processes followed. But they assumed that (i) *the second moments* (in the subcritical case) *or the third moments* (in the critical case) *are finite* and (ii) *the mean matrix is positively regular*. Joffe and Spitzer [9] obtained the results for discrete time processes without the hypothesis (i), and Sevastyanov [14] extended them for continuous time processes. Their results are final for the processes satisfying the condition (ii). However, when the condition (ii) fails, somewhat different phenomena occur. Chistyakov [3] illustrated it for the continuous time subcritical processes with the hypothesis (i). For the continuous time critical processes, the results of Savin and Chistyakov [12] for the processes with three particle types and the hypothesis (i) are very suggestive.

In this paper, we shall give the whole asymptotic behavior of discrete and continuous time multitype Galton-Watson processes without the hypotheses (i) and (ii) (but with some weaker hypotheses). The processes are decomposed into elementary subprocesses. When the elementary subprocesses have positively regular mean matrices, the results naturally coincide with those of [8], [9], [10] and [14]. But when they are reducible, the rate that the generating functions