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The behavior of solutions of some non-linear diffusion equations for large time

By

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0. Introduction

Consider the semi-linear diffusion equation

$$u^{\cdot}=\tfrac{1}{2}u''+F(u) \qquad t>0,\ -\infty<x<\infty \tag{1}$$

$$(u=u(t, x),\quad u^{\cdot}=\partial u/\partial t,\quad u''=\partial^2 u/\partial x^2)$$

with the initial condition

$$u(0, \cdot)=f. \tag{2}$$

The function F is always assumed in this paper to satisfy

$$F\in C^1[0, 1],\quad F(0)=F(1)=0 \quad \text{and} \quad F(u)>0 \quad 0<u<1 \tag{3}$$

and the initial function f to be measurable and compatible to F, i.e. $0\leqq f\leqq 1$. Our interest is in the behavior of the solution for large time t.

We mean by the *solution* of (1) and (2) such a function $u(t, x)$ defined on the upper half plane $[0, \infty)\times(-\infty, \infty)$ that (i) $0\leqq u\leqq 1$, (ii) u has continuous derivatives $u^{\cdot}$ and u'' and satisfies (1) in $(0, \infty)\times(-\infty, \infty)$, and (iii) $u(t, \cdot)$ converges to f as $t\downarrow 0$ in locally L^1 sense. It is well known that such a solution exists and is unique. We denote it by $u(t, x; f)$. It is clear that $u(t, x; u(s, \cdot\,; f))=u(t+s, x; f)$ (Huygens property) and $u(t, x; f(\cdot+y))=u(t, x+y; f)$. We sometimes consider the equation (1) with different F's and in such cases use the notation $u(t, x; f; F)$ in order to elucidate the dependence on F. There are just two trivial solutions of (1): $u\equiv 0$ and $u\equiv 1$. We always consider our problem for non-trivial initial functions f; $f\not\equiv 0$ and $\not\equiv 1$. Such initial functions are called *data*. We will mainly deal with such data that $f(x)\to 0$ as $x\to\infty$.

The behavior of a solution $u(t, x; f)$ is closely related to solutions of ordinary differential equations

$$\tfrac{1}{2}w''+cw'+F(w)=0 \qquad (0\leqq w\leqq 1) \tag{4}$$

where c is a real constant. This equation is formally obtained if we substitute the